Incentive Schemes for p2p

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p2p and public goods

- Public good:
  - non-rivalrous (one peer's consumption does not reduce the amount available to others)
  - positive externalities (a peer benefits from the presence of other peers because of cost sharing)
- p2p: content, coverage, connectivity have PG aspects
- Major problem: free-riding
- Our goal: design optimal incentives for contribution

Public Goods

- Non-excludable and non-rival goods
- Incentive problem in provisioning: the free-rider problem

Example: provision a common facility of size = 1,2

$$u_i(1) = 2, u_i(2) = 4, c_i(1) = 3$$

Player B

Player A		provision 1	provision 2
	provision 1	1,1	-1,2
	provision 2	2,-1	0,0

Free riding: player i prefers the other player to contribute

Free market fails to provision optimum amount of public goods

Incentives in p2p

- P2p systems exhibit strong public good aspects (externalities)
- Implication: "free market" solution is inefficient
  - each peer maximizes own net benefit
  - actions affect others
  - hence private optimum differs from social optimum
- Need regulation: use prices or rules to influence behaviour
  - incentives for each peer reflect the effect it has on others
  - example of a rule: downloads = uploads

The role of information

- Problem: optimal design requires information on user types
  - under full info: personalized price/rule for each peer
  - “first-best” policy
  - Existing approaches based on heuristics
    - reciprocity based punishments/rewards
- How can the system/planner/network manager get the required information to design optimal contribution rules?
  - if lucky, can gather personalized data about users
  - otherwise, users **must be given incentives** to reveal relevant information to planner
- Mechanism Design: set prices/rules to encourage users to act truthfully, maximize social welfare
  - for large n, use simple rules!

Incomplete Information

A non-excludable public good

- n agents bargain to provision a public good
- Q = quantity of public good, all agents enjoy it
- $c(Q)$ = cost of public good, agent i pays p_i

$$\theta_i u(Q) - p_i$$ = agent's i net benefit

- θ_i iid, has distribution F

- Examples:

$$u(Q) = Q^{1/2}, \quad c(Q) = Q^2$$

$$Q \in \{0,1\}, \quad u(Q) = Q, \quad c(Q) = cQ$$

Allocations

- For each $\theta = (\theta_1, \ldots, \theta_n)$
  - what quantity $Q(.)$?
  - what contributions $p_1(.), \ldots, p_n(.)$?
- Feasible: $c(Q(\theta)) \leq \sum p_i(\theta)$
- incentive compatible: $E_{\theta_{-i}}[\theta_i u(Q(\theta_i, \theta_{-i}) - p_i(\theta_i, \theta_{-i})]$

  $\geq E_{\theta_{-i}}[\theta_i u(Q(\hat{\theta}_i, \theta_{-i}) - p_i(\hat{\theta}_i, \theta_{-i})]$
- Individually rational: $E_{\theta_{-i}}[\theta_i u(Q(\theta_i, \theta_{-i}) - p_i(\theta_i, \theta_{-i})] \geq 0, \quad \forall \theta_i$

Allocations (2)

- First-best: maximizes Social Welfare (SW) under complete information (is trivially feasible)

$$\max_{Q(.)} \sum \theta_i u(Q(\theta)) - c(Q(\theta))$$
$$= \max_{Q} u(Q) \sum \theta_i - c(Q)$$

- Second-best: maximizes SW under incomplete information, i.e.,
  - subject to
    - feasibility
    - incentive compatibility
    - individual rationality

Example

Build a bridge

$$Q \in \{0,1\}, \quad u(Q) = Q, \quad c(Q) = \overset{\le 1}{c}Q, \quad \theta_i \text{ iid uniform on } [0,1]$$

- First-best policy:

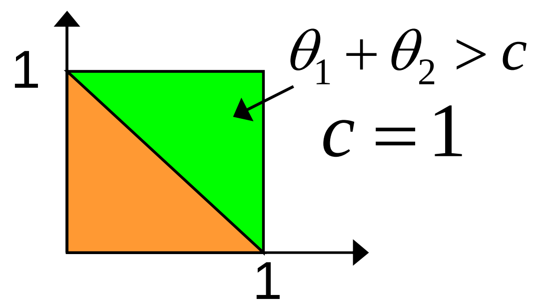
$$\max_{Q(.)} \sum \theta_i u(Q(\theta)) - c(Q(\theta))$$

- Solution (n=2):

$$Q(\theta) = 1 \text{ if } \theta_1 + \theta_2 > c, \text{ use any } p_1 \le \theta_1, p_2 \le \theta_2, \text{s.t. } p_1 + p_2 = c$$

$$Q(\theta) = 0 \text{ if } \theta_1 + \theta_2 \le c$$

$$p_i(\theta) = \frac{\theta_i}{\theta_i + \theta_j} c$$



Example (2)

- Why should agents declare their actual θs ?
- If $p_i(\theta) = \dfrac{\theta_i}{\theta_i + \theta_j} c$ then agent with highest θ_i gains by declaring less -> SW loss
- Which is the best allocation policy? (second-best)
- Impossibility Theorem (Myerson-Satterhwaite (1983))
  - Second Best (SB) < First Best (FB)

More on second best policies

- Problem:

$$\max_{Q(.),\ p_1(.), \ldots, p_n(.)} E\left[\sum \theta_i u(Q(\theta)) - c(Q(\theta))\right]$$

- subject to
  - feasibility $\quad c(Q(\theta)) \le E\left[\sum p_i(\theta)\right]$
  - individual rationality $\quad E_{\theta_{-i}}[\theta_i u(Q(\theta_i, \theta_{-i}) - p_i(\theta_i, \theta_{-i})] \ge 0, \quad \forall \theta_i$
  - incentive compatibility ….

A lemma for IC

- Let $V_i(\theta_i) = E_{\theta_{-i}}[u(Q(\theta_i,\theta_{-i}))], \quad P_i(\theta_i) = E_{\theta_{-i}}[p_i(\theta_i,\theta_{-i})]$

- A necessary and sufficient condition for IC is

$$P_i(\theta_i) = P_i(0) + \theta_i V_i(\theta_i) + \int_o^{\theta_i} V_i(y)dy$$

- Given IC, the system is IR iff

$$P_i(0) \leq 0$$

- Then $E_\theta[\sum p_i(\theta)] = E_\theta[\sum P_i(\theta_i)]$

$$= E_\theta[\sum_1^n (\theta_i - \frac{1-F(\theta_i)}{f(\theta_i)})u(Q(\theta))]$$

The SB problem

- Solve

$$\max_{Q(.)} \; E_{\theta}[\sum_{1}^{n} \theta_i u(Q(\theta)) - c(Q(\theta))]$$

- subject to

$$E_{\theta}[\sum_{1}^{n} (\theta_i - \frac{1 - F(\theta_i)}{f(\theta_i)}) u(Q(\theta)) - c(Q(\theta))] \geq 0$$

The Lagrangian

- A Lagrangian formulation

$$\int [\sum_{1}^{n} \theta_i u(Q(\theta)) - c(Q(\theta))]\, dP_n \qquad\qquad g(\theta)$$

$$+ \lambda \int [\sum_{1}^{n} (\theta_i - \frac{1 - F(\theta_i)}{f(\theta_i)}) u(Q(\theta)) - c(Q(\theta))]\, dP_n$$

- Calculation of Q(.): point wise maximization

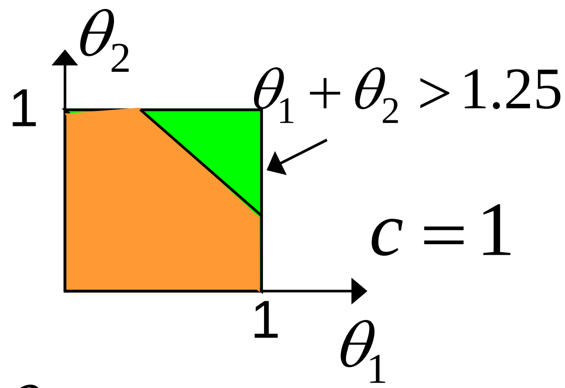
$$Q(\theta) =$$

$$\arg \max_{Q} \left[\sum_{1}^{n} \theta_i u(Q) - c(Q) + \lambda \left(\sum_{1}^{n} g(\theta_i) u(Q) - c(Q) \right) \right]$$

Back to the bridge construction

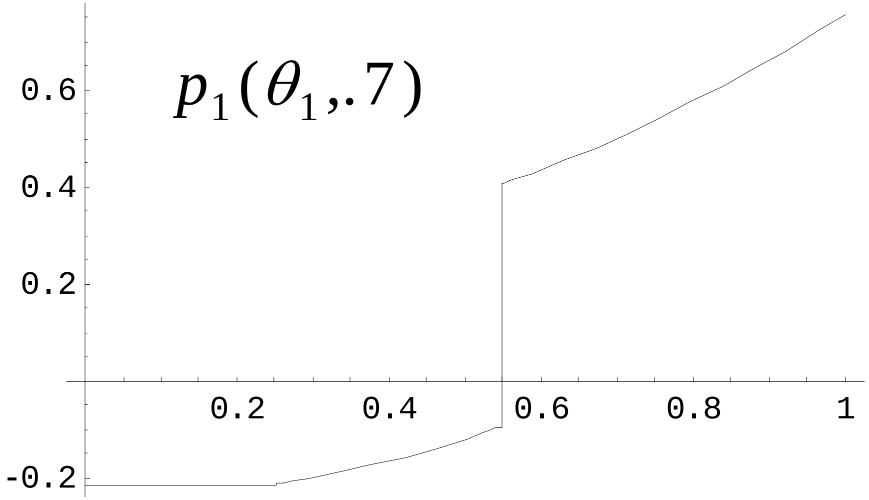
- Simple calculation of optimal Q(.)

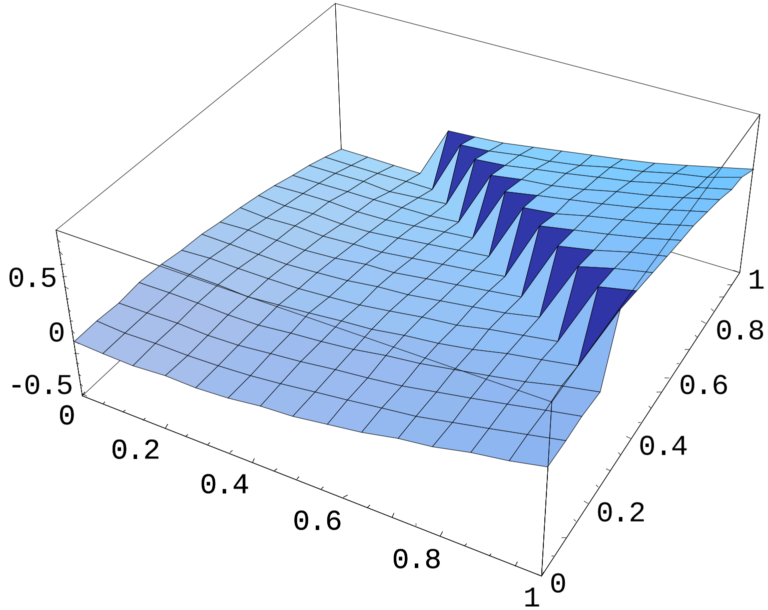
$$Q(\theta) = 1 \text{ iff } \sum_{1}^{n} \theta_i - \frac{\lambda}{1+\lambda} \sum_{1}^{n} \frac{1 - F(\theta_i)}{f(\theta_i)} \geq c$$

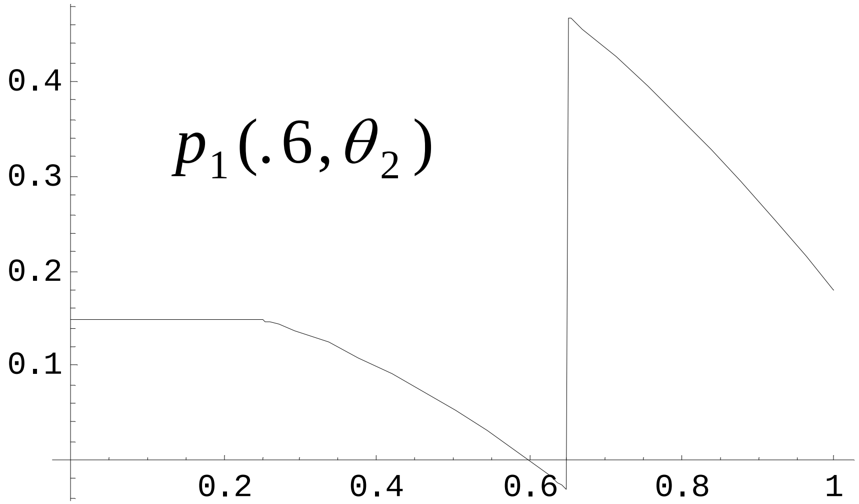


- Incentive payments: extremely complex!
  - functions of complete vector $\theta_1, \ldots, \theta_n$
  - involve money transfers between agents
  - no known simple approximation

The payments for n=2







$$Q = 1 \text{ if } \theta_1 + \theta_2 \geq 1.25$$

Some remarks

- Optimal incentive policies are impractical to evaluate in most situations
  - Need for good approximations
- Existing results for specific models suggest that as $n \to \infty$

$$\frac{SB}{FB} \to 0$$

- If **exclusions** are possible, then

$$\frac{SB}{FB} \to \alpha > 0$$

  - Incentive payments converge to fixed contributions
  - can we obtain a general theorem?

Exclusions

- Part of the allocation policy is the exclusion capability

Agent i is excluded if $\pi_i(\theta_1,\ldots,\theta_n)=0$

Agent i participates if $\pi_i(\theta_1,\ldots,\theta_n)=1$

- Second-Best policy: solve

$$\max_{Q(.),\ \pi_1(.),\ldots,\ \pi_n(.)} E[\sum_1^n \theta_i \pi_i(\theta) u(Q(\theta)) - c(Q(\theta))]$$

such that

$$E_\theta[\sum_1^n \pi_i(\theta) g(\theta_i) u(Q(\theta)) - c(Q(\theta))] \geq 0$$

A limit theorem

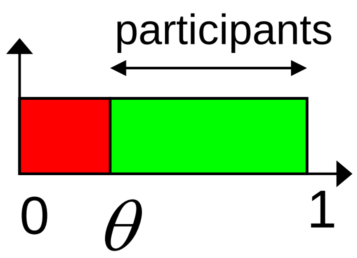
Suppose

- $u(Q) = AQ^{\alpha}$, and $c(n,Q) = Bh(n)Q^{\beta}, \beta \geq 1 > \alpha$
- and Q^*, θ^* maximize

$$P = \max_{\theta \in [0,1], Q \geq 0} nu(Q) \int_{\theta}^{1} yf(y)dy - c(n,Q)$$

subject to

$$\underbrace{n[1 - F(\theta)]}_{\text{\# of participants}} \underbrace{\theta u(Q)}_{\text{fixed fee}} - c(n,Q) \geq 0$$



Then the simple mechanism $\pi_i(\theta) = 1\{\theta_i \geq \theta^*\}, \; Q(\theta) = Q^*,$

$p_i(\theta) = \theta^* u(Q^*)$, achieves $P \leq SB \leq (1 + O(n^{-1/5}))P$

Why large systems are simpler

- Why size helps?
  - in a large network it is hard to get people pay more than a minimum
- As the number of peers gets larger
  - a peer feels that his own declaration will have a negligible effect on the final system size
  - hence his strongest incentive is to only reduce his payment
  - therefore he declares the minimum possible theta which corresponds to the minimum fixed fee by agreeing to participate.

Example

$$u(Q) = 0.6Q^{1/2}, \quad c(Q) = Q, \quad \theta_i \text{ uniform in } [0,1]$$

$$\max_{\theta \in [0,1], Q \geq 0} \left(n \int_\theta^1 y dy \right) 0.6Q^{1/2} - Q$$

$$s.t. \quad n[1-\theta]\theta \; 0.6Q^{1/2} - Q \geq 0$$

Solution: $\theta^* = 1/4, \quad Q^* = 0.0126n^2, \quad SW = 0.006328n^2$

- satisfaction of cost coverage constraint: reduction of SW by 43%

Applications

- File Sharing
  - public good = content availability
  - that is, number of total distinct files shared
- P2P WLANs
  - peers share wireless access to the internet
  - public good = coverage

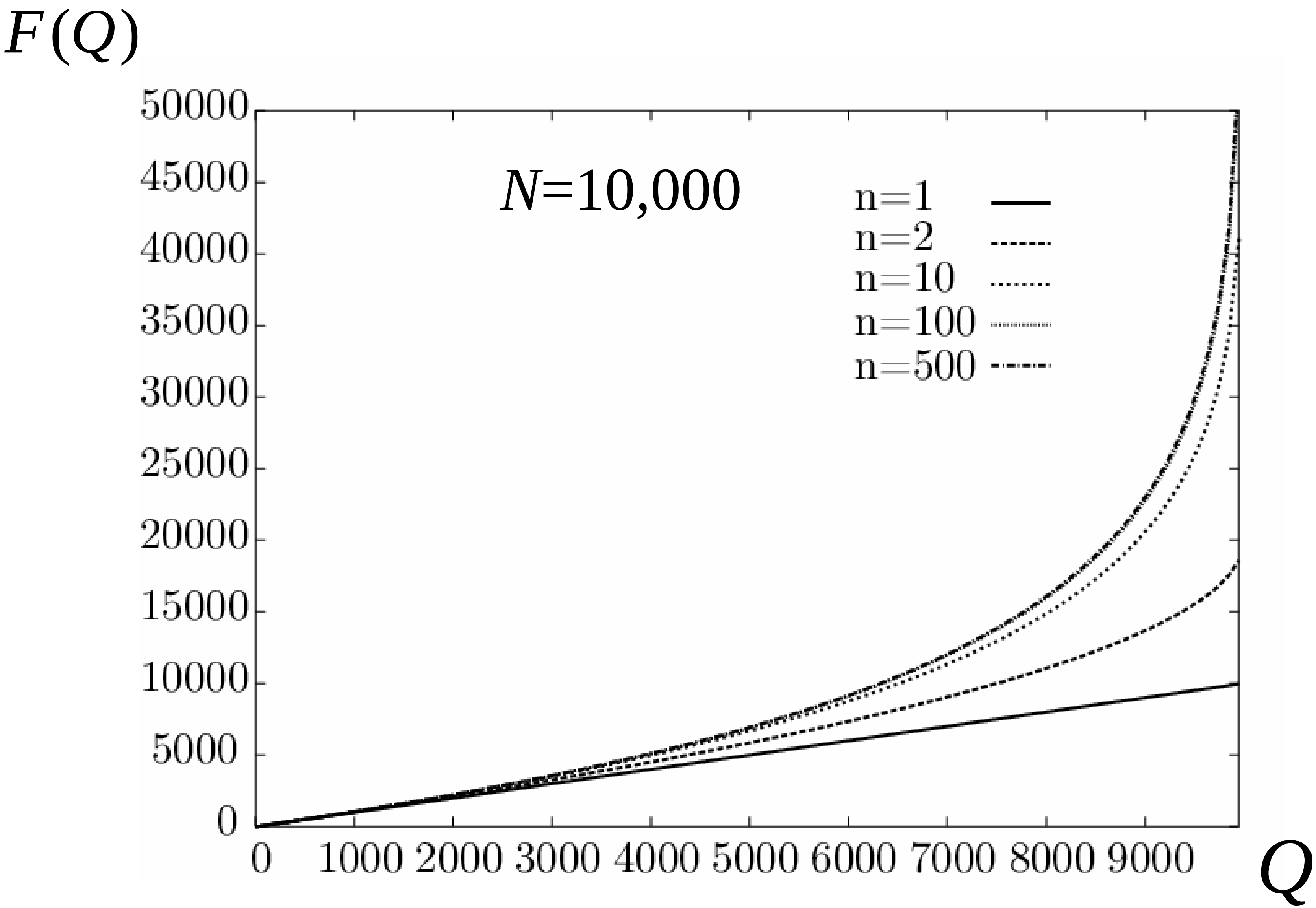
File sharing

- Q : expected number of distinct files
- peer i :
  - utility = $\theta_i u(Q)$,
  - cost = f_i = number of files provided to the system
  - f_i randomly chosen from N files
- Rewrite equations in terms of F

$$Q(F) \approx N(1 - e^{-F/N}), \text{ where } F = \sum f_i$$

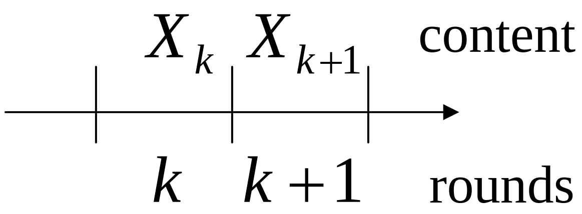
- Compute optimum fixed contributions as before

The function *F(Q)*

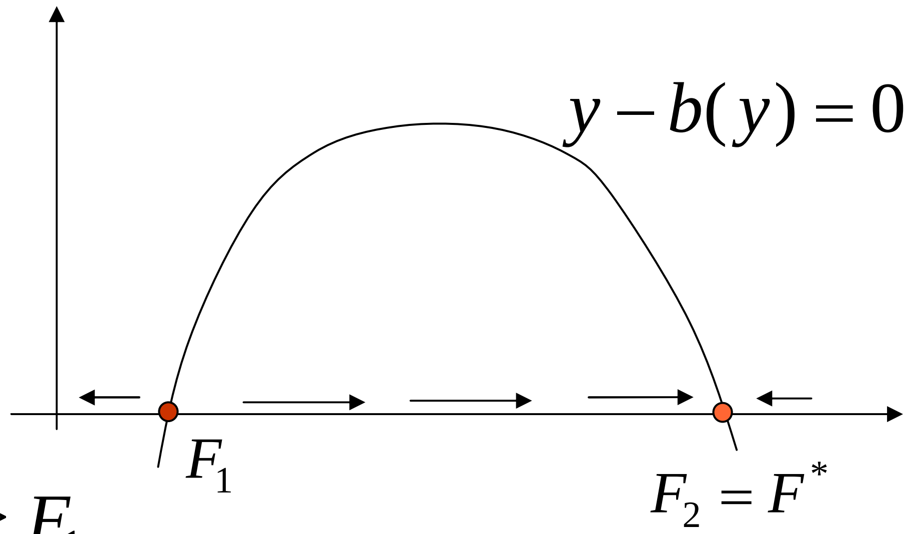


Stability

- Assume contribution f^* fixed
- Participation decision: based on prior expectation regarding total content availability *F*
- Will F^* be reached?



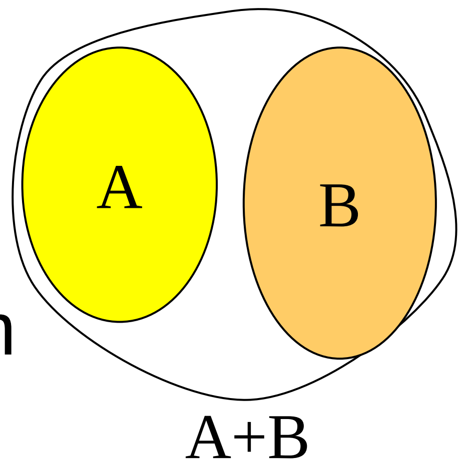
$$X_{k+1} = b(X_k, f^*)$$



stability if $X_0 \geq F_1$

Group formation (1/3)

- Assume peers of different sub-types
- Type A: $\theta_i^A \sim [0,0.5]$ (e.g. ISDN users)
- Type B: $\theta_i^B \sim [0.5,1]$ (e.g. DSL users)
- Do they gain more by
  - forming 2 distinct groups vs forming a larger group?
  - being distinguished by the system in the larger group?



Group formation (2/3)

- Group A: $\theta_i^A \sim [0,0.5]$ (e.g. ISDN users)
- Group B: $\theta_i^B \sim [0.5,1]$ (e.g. DSL users)

Assume that the percentage of each group in the mix is 50% (n=1000)

Welfare	Group A	Group B	Total
Distinct groups	3296	35156	38452
Indistinguishable	6976 (+ 111%)	44792 (+ 27%)	51768
Distinguishable	31249 (+ 848%)	31250 (-11%)	62500

Group formation (3/3)

- How to provide better incentives for both types to combine and reveal their types?
  - reduce cost of heavy users by limiting upload rates
  - reduce fees of heavy users
- Offer sets of tariffs (versioning)
  - allow self-selection
- Model difference in cost for uploading
  - higher-cost peers benefit in a larger group when types can be distinguished

Heterogeneous file popularity

- General case: $u(F_1, F_2), c(F_1, F_2)$
  - specify contributions f_1^*, f_2^*

F_1 : popular content

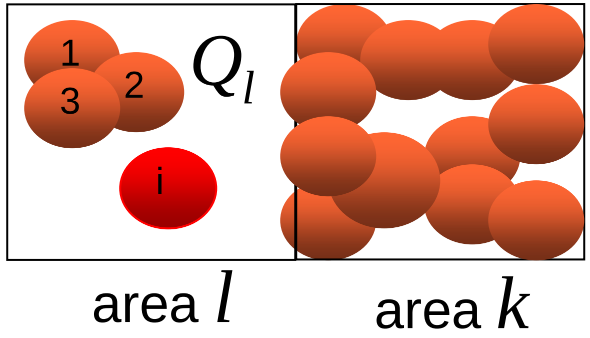
F_2: less popular content

- Interesting case: $u(aF_1 + F_2),\ c(F_1, F_2) = bF_1 + F_2$
- Then, provide both types only if $a = b$
- Optimum contribution is a scalar f^*
  - a peer can provide any combination $f_1, f_2,\ s.t.\ af_1 + f_2 = f^*$
  - measuring rate of uploads is a good proxy

Adaptation

- What if $H(\cdot)$ not known?
- In general incentive to shade declarations
- Repeated game formulation: in each round, peer *i* samples θ_i from *H* , declares θ_i
    - truth-telling equilibrium

WLANs



- L geographic areas
  - coverage: fraction of area covered by hotspots
  - area l has coverage Q_l
- Peer i of area l
  - obtains benefit $\theta_i^l u^l(Q_1, \ldots, Q_L)$
  - contributes payment (coverage) f_i^l
- Feasibility: $Q_l \leq \sum_{i=1,n_l} f_i^l, \quad \text{for all areas } l$
- Problem: find optimum incentive scheme, maximize efficiency
- Our Theorem holds
  - use fixed contribution schemes

Conclusions

- In p2p systems with strong PG aspects
  - Fixed contribution schemes can be efficient
  - Result to tractable optimization problems
  - Provide useful insight to many interesting questions
  - Information regarding user types may be strategic
- Open issues:
  - more complex cost structures
  - adaptation
  - multiple round games
  - use existing data to tune economic model

Complete Information

An economic model of peering

Net utility of peer *i* : $u_i(r, f) = b_i(r_i, \sum_{j=1}^{N} f_j) - c_i(f_i, \sum_{j=1}^{N} r_j)$

Equilibrium strategy: each peer solves

$$\max_{r_i, f_i} b_i(r_i, \sum_{j=1}^{N} f_j) - c_i(f_i, \sum_{j=1}^{N} r_j) \Rightarrow f_i \approx 0$$

r_i : resource request rate of peer *i*

f_i : resources contributed by peer *i*

How do we achieve efficiency?

- provide incentives
- traditional approach: use Lindahl prices
- Lindahl price represents total externality imposed by an individual peer
- hence it is personalized
- can achieve full efficiency with these prices

2 problems with Lindahl prices

- informationally very demanding
- this can be relaxed in a large network: personalized prices can be approximated by a uniform price
- payments present difficulties in a large, anonymous network with many small transactions
- Use rules instead of prices

Maximizing efficiency

$$S \equiv \max_{r,f} \sum_{i=1}^{N} \left[b_i(r_i, \sum_{j=1}^{N} f_j) - c_i(f_i, \sum_{j=1}^{N} r_j) \right] \Rightarrow r_i^*,\ f_i^*$$

<u>Lindahl prices:</u> Peer *i* solves

$$\max_{r_i, f_i} \left[b_i(r_i, f^*) - p_i^r r_i + p_i^f f_i - c_i(r^*, f_i) \right] \quad s.t. \quad p_i^r = \sum_{j \neq i} \left. \frac{\partial c_j(r, f_j)}{\partial r} \right|_{\{r_i^*, f_i^*\}}$$

$$p_i^f = \sum_{j \neq i} \left. \frac{\partial b_j(r_i, f)}{\partial f} \right|_{\{r_i^*, f_i^*\}}$$

<u>Rules:</u> Peer *i* solves

$$\max_{r_i, f_i} \left[b_i(r_i, f^*) - c_i(r^*, f_i) \right] \quad s.t.\ r_i \le \beta_i f_i + a_i$$

Interesting results

- For large N uniform prices, but not uniform rules
- Stability of rules
- Practical perspective
  - heuristics to approximate optimal prices and rules for mixed groups using information from single-type groups

Optimal prices

$$\max_{r_i, f_i} \left[b_i(r_i, f^*) - p_i^r r_i + p_i^f f_i - c_i(r^*, f_i) \right]$$

$$p_i^r = \sum_{j \neq i} \left. \frac{\partial c_j(r, f_j)}{\partial r} \right|_{\{r_i^*, f_i^*\}}$$

Pay for the cost incurred to others due to requests

$$p_i^f = \sum_{j \neq i} \left. \frac{\partial b_j(r_i, f)}{\partial f} \right|_{\{r_i^*, f_i^*\}}$$

Get paid for the benefit offered due to files shared

As *N* gets large non-uniform optimal prices can be approximated by uniform ones (account for total cost and benefit)

$$p^r = \sum_{j=1}^{N} \left. \frac{\partial c_j(r, f_j)}{\partial r} \right|_{\{r_i^*, f_i^*\}} \qquad p^f = \sum_{j=1}^{N} \left. \frac{\partial b_j(r_i, f)}{\partial f} \right|_{\{r_i^*, f_i^*\}}$$

Proof (1)

- <u>Assumption 1 (Bounded heterogeneity)</u>

$\exists\ \overline{b}, \underline{b}, \overline{c}, \underline{c},$ where $0 < \underline{x} \leq \overline{x} < +\infty, x \in \{b, c\},$ s.t.

$$\sup_{f \in R_+, r \in R_+} \frac{\partial b_i(r, f)}{\partial f} \in [\underline{b}, \overline{b}],$$

$$\sup_{f \in R_+, r \in R_+} \frac{\partial bc_i(r, f)}{\partial r} \in [\underline{c}, \overline{c}]$$

- And let $p^r = \sum_{j=1}^{N} \frac{\partial c_j(r, f_j)}{\partial r}\bigg|_{\{r_i^*, f_i^*\}}$ $p^f = -\sum_{j=1}^{N} \frac{\partial b_j(r_i, f)}{\partial f}\bigg|_{\{r_i^*, f_i^*\}}$

- We will prove that for given $\overline{b}, \underline{b}, \overline{c}, \underline{c}$, and for a constant $\varepsilon > 0$, there exists a critical number of $\overline{N}$ peers such that for $N > \overline{N}$

$$\frac{\left|p_i^x - p^x\right|}{p^x} < \varepsilon,\ x \in \{r, f\},\ i = 1, \dots, N$$

Proof (2)

- We will prove the result for p^r and for peer i
- Note that $\left|p_i^r - p^r\right| = \dfrac{\partial c_i(\sum_k r_k, f_i)}{\partial r_i}$
- Hence $\left|p_i^r - p^r\right| \le \bar{c}$ and in addition $p^r \ge N\underline{c}$
- Therefore $$\frac{\left|p_i^r - p^r\right|}{p^r} \le \frac{\bar{c}}{N\underline{c}}$$
- Define $\overline{N} \equiv \bar{c} / \varepsilon\underline{c}$. By assumption 1, for N sufficiently large, $N > \overline{N}$

  and so $$\frac{\left|p_i^x - p^x\right|}{p^x} < \varepsilon$$

Optimal rules

$$\max_{r_i, f_i}\left[b_i(r_i, f^*) - c_i(r^*, f_i)\right] \qquad s.t.\ r_i \le \beta_i f_i + a_i$$

$$\max_{r_i, f_i}\left[b_i(a_i + \beta_i f_i, \sum_j f_j) - c_i(a_i + \beta_i f_i + \sum_{j \ne i} r_j, f_i) \right]$$

$$\beta_i \approx \frac{p^f}{p^r}$$

$$a_i = r_i^* - \beta_i f_i^*$$

Optimal rules may need personalization!

Stability of Rules

We assume that as a function of time, $\dot{f}_i = \phi_i$, and choose as a potential Liapunov function $V = \sum_i \phi_i^2$. Now[6],

$$\dot{\phi}_i = \frac{\partial^2 b_i}{\partial r^2}\beta_i^2\phi_i + \sum_{j\neq i}\frac{\partial^2 b_i}{\partial r \partial f}\beta_i\phi_j - \frac{\partial^2 c_i}{\partial f^2}\phi_i - \sum_{j\neq i}\frac{\partial^2 c_i}{\partial r\partial f}\beta_j\phi_j\,.$$

Then

$$\dot{V} = \sum_i 2\phi_i\dot{\phi}_i = 2\sum_i[\frac{\partial^2 b_i}{\partial r^2}\beta_i^2 - \frac{\partial^2 c_i}{\partial f^2}]\phi_i^2 + \sum_i\sum_{j\neq i}[\frac{\partial^2 b_i}{\partial r\partial f}\beta_i - \frac{\partial^2 c_i}{\partial r\partial f}\beta_j]\phi_i\phi_j\,.$$

Let

$$A_{ii} = \frac{\partial^2 b_i}{\partial r^2}\beta_i^2 - \frac{\partial^2 c_i}{\partial f^2},\ A_{ij} = \frac{\partial^2 b_i}{\partial r\partial f}\beta_i - \frac{\partial^2 c_i}{\partial r\partial f}\beta_j\,.$$

Then

$$\begin{aligned}
\frac{1}{2}\dot{V} &= \sum_i A_{ii}\phi_i^2 + \sum_i\sum_{j\neq i}A_{ij}\phi_i\phi_j \\
&\leq -\min|A_{ii}|\sum_i\phi_i^2 + \max|A_{ij}|\sum_i\sum_{j\neq i}\phi_i\phi_j \\
&\leq -\min|A_{ii}|\sum_i\phi_i^2 + \max|A_{ij}|(N-1)\sum_i\phi_i^2\,.
\end{aligned}$$

Hence, a sufficient condition for stability is

$$\min|A_{ii}| > (N-1)\max|A_{ij}|\,.$$

Heuristics

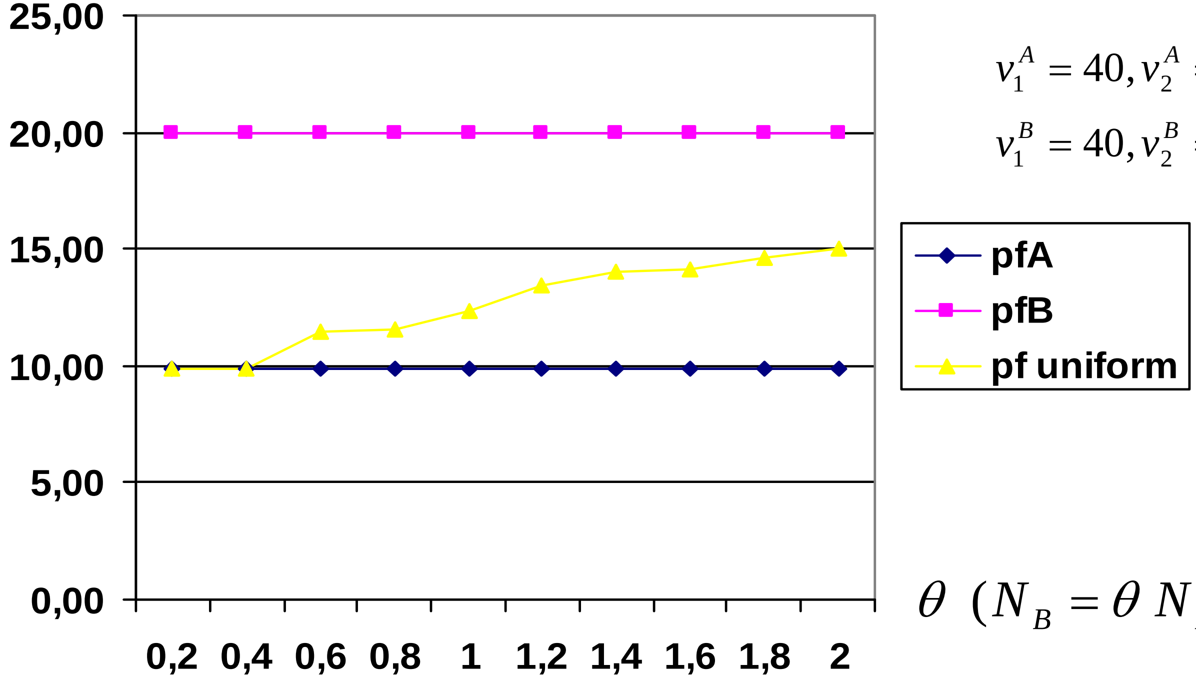
- We could use heuristics to approximate optimal prices and rules for mixed groups using information from single-type groups
- Simple example: specific net benefit function
  - $$b_i = v_1 \log r_i + v_2 \log \sum_j f_j - k_1 f_i - k_2 \left(\frac{f_i}{\sum_j f_j} r_j\right)^2$$
  - Optimal prices independent from N
- Calculate the optimal prices of homogeneous groups
- For a 2-type case, calculate the optimal uniform prices and observe their relation with the homogeneous ones

First Results

- Uniform prices lie always between the homogeneous ones
  - The same with uniform (but not optimal) rules
- The exact distance depends on
  - The percentage of peers in the mixed group
  - The different types
- Welfare loss depends on heterogeneity

Prices (1)

The uniform price of the mixed group depending on N_A, N_B



$v_1^A = 40, v_2^A = 40, k_1^A = 5, k_2^A = 0.1$

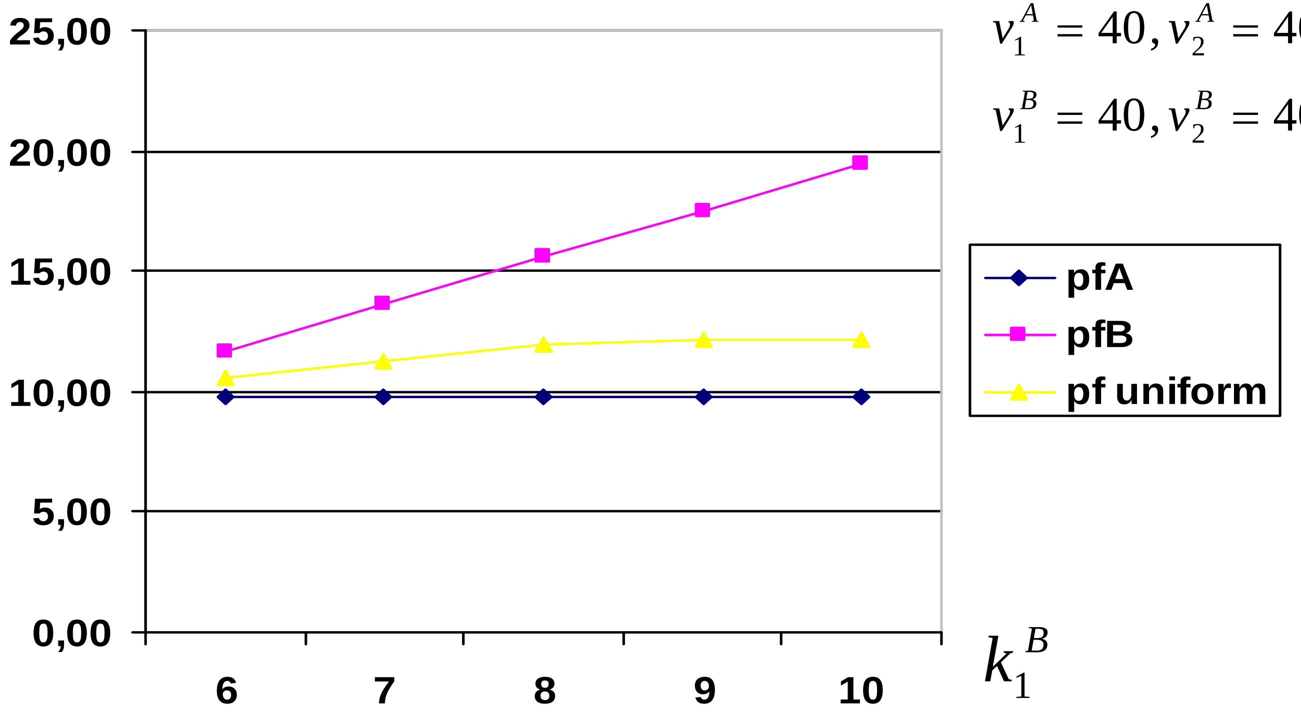
$v_1^B = 40, v_2^B = 40, k_1^B = 10, k_2^B = 0.1$

$\theta \ (N_B = \theta N_A)$

Simple example: $b_i = v_1 \log r_i + v_2 \log \sum_j f_j - k_1 f_i - k_2 (\frac{f_i}{\sum_j f_j} r_j)^2$, 2 types (A and B)

Prices (2)

The uniform price of the mixed group depending on peer types

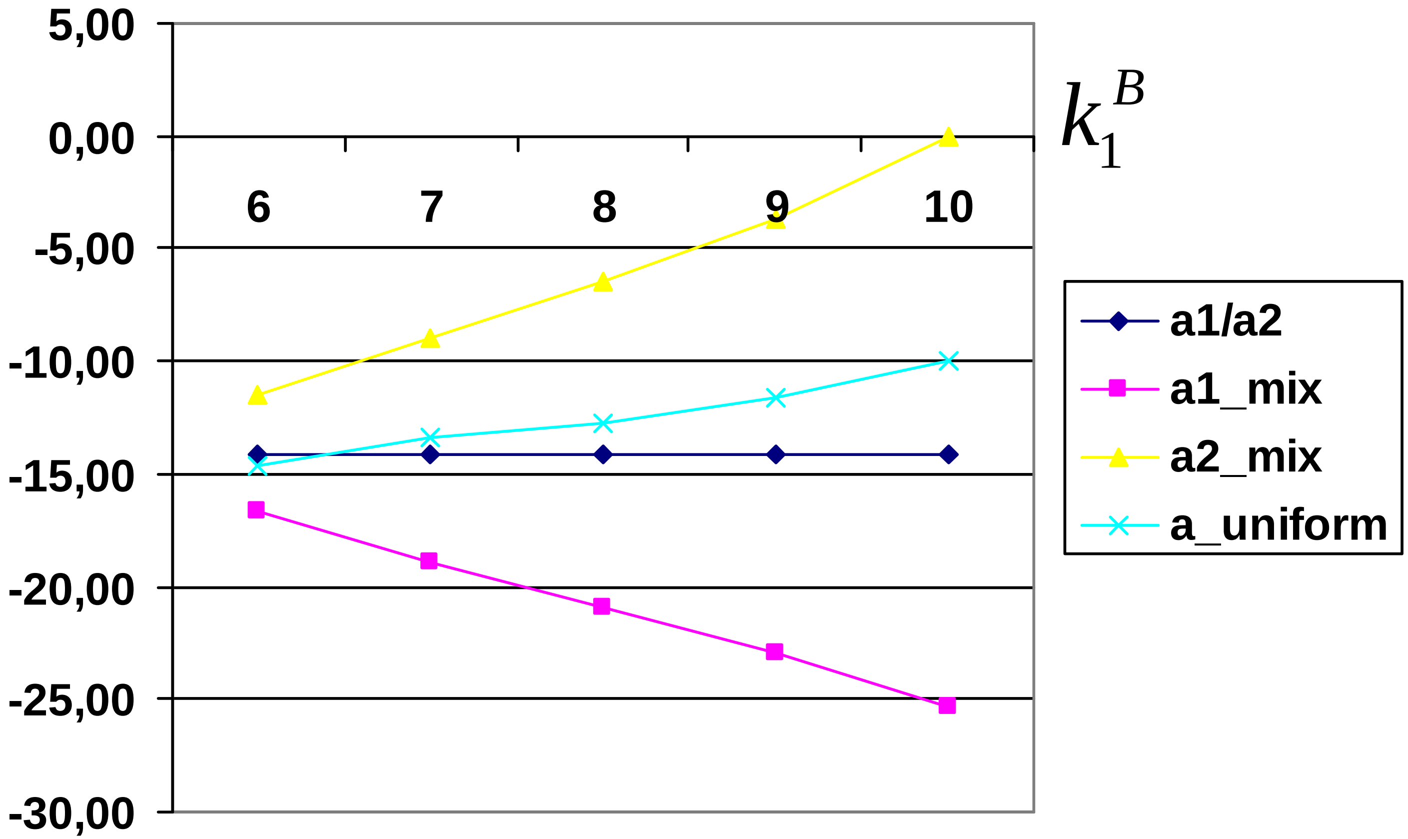


$$v_1^A = 40, v_2^A = 40, k_1^A = 5, k_2^A = 0.1$$

$$v_1^B = 40, v_2^B = 40, k_2^B = 0.1$$

k_1^B

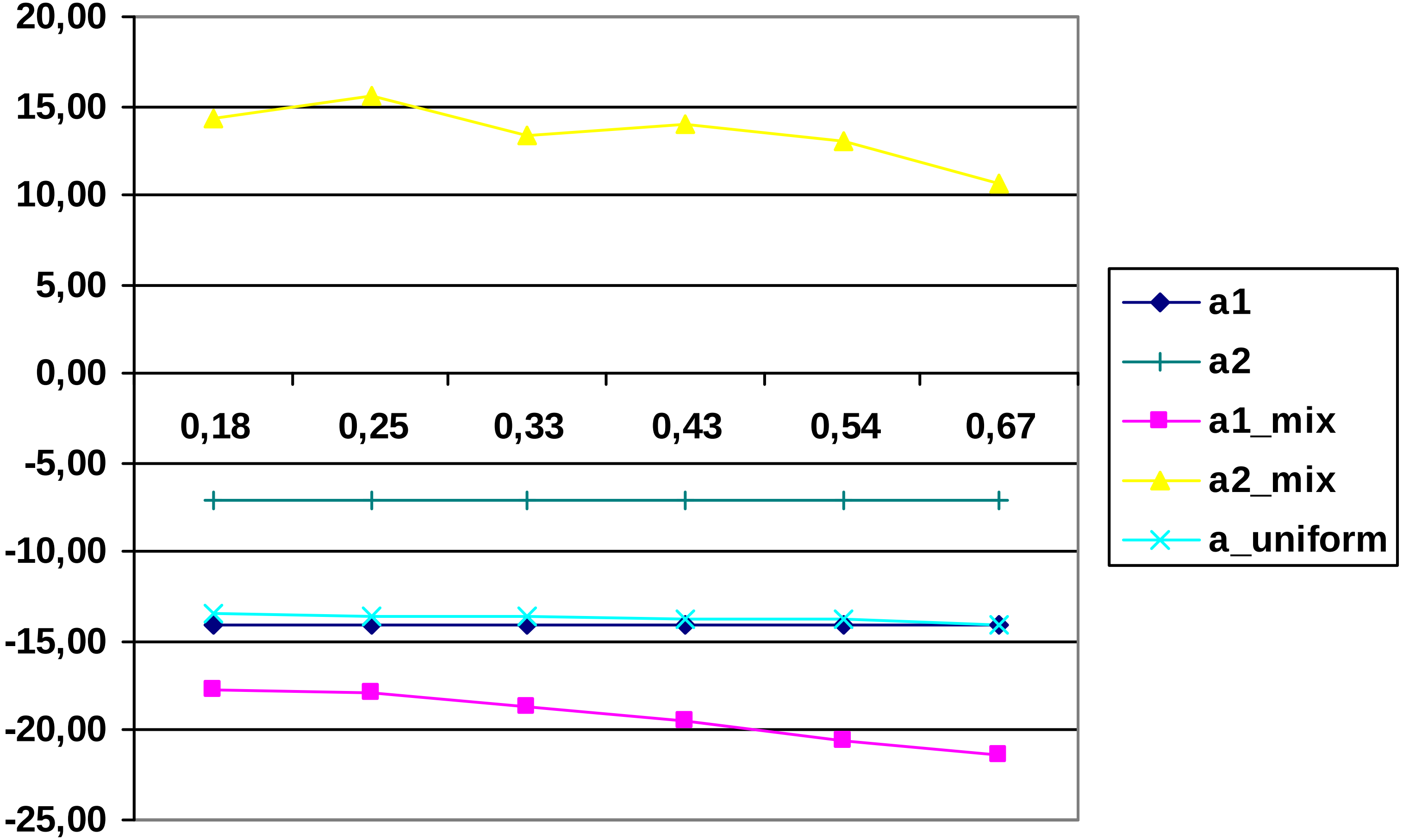
Rules (1)



$$v_1^A = 40, v_2^A = 40, k_1^A = 5, k_2^A = 0.1$$

$$v_1^B = 40, v_2^B = 40, k_2^B = 0.1$$

Rules (2)



$$v_1^A = 40, v_2^A = 40, k_1^A = 5, k_2^A = 0.1$$

$$v_1^B = 160, v_2^B = 40, k_1^B = 5, k_2^B = 0.1$$

Complete vs incomplete information

- Can we compare the efficiency of the various mechanisms in the context of complete and incomplete information?
- Need to use a simple model!
  - one unknown parameter

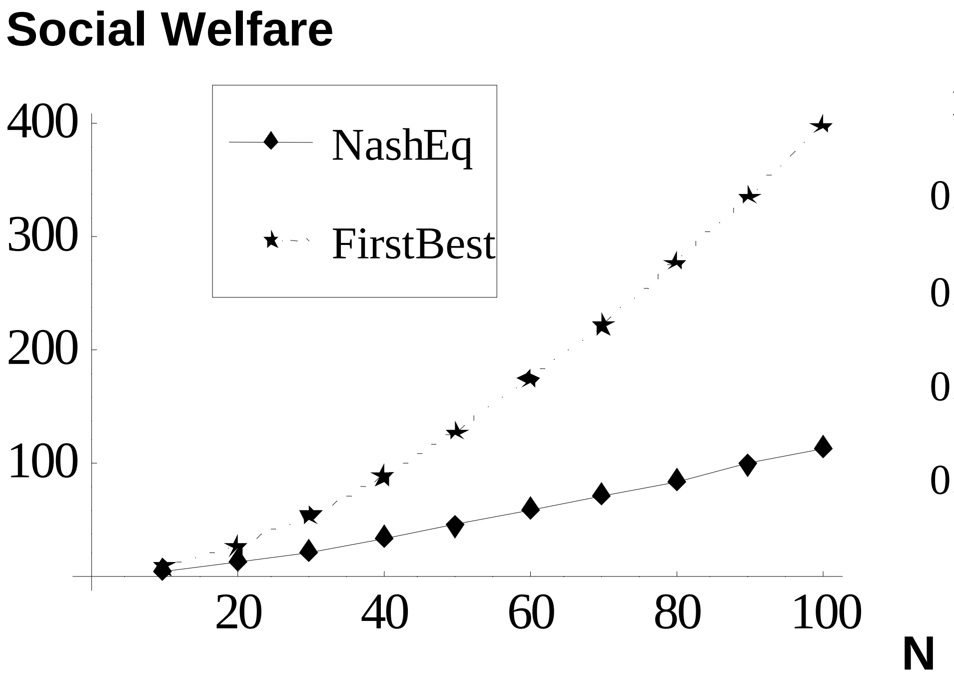
A single parameter model for file sharing

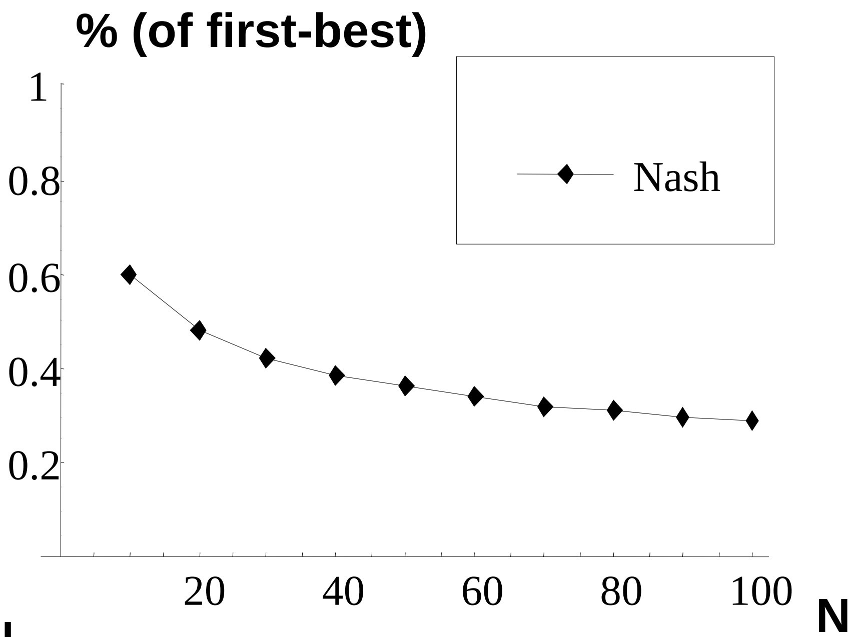
Payoff of agent *i* : $u_i(\mathrm{f}) = \theta_i \upsilon(\sum_{j=1}^{N} \sqrt{f_j}) - f_i$

- Each peer decides on the number of files f_i that it shares
- Cost function is linear and the same for everyone
- Peers differ in the payoff parameter θ_i (drawn from distribution *F*)
- Benefit function υ depends on the sum of the square roots of files shared expressing file duplication

Nash Equilibrium vs Social Optimum

All peers maximize their payoff: $\max_{f_i} u_i(\mathrm{f})$, taken as given the files shared by all other peers





Peers have the incentive to free-ride on efforts of others

Complete Information Mechanisms

***First-best rule**: <u>Enforce</u> all peers to contribute* f^* *files*

***First-best price**: Peer* i *is paid with* p_i *per file shared*

Rules with participation incentives

$$\tilde{f}_i = \begin{cases} \theta_i \upsilon(\tilde{F}), & i < k \\ \theta_k \upsilon(\tilde{F}), & i \geq k \end{cases}$$

Uniform prices

$$p = 1 - \frac{\max\{\theta_1, \ldots, \theta_N\}}{\Theta}$$

Simulation assessment

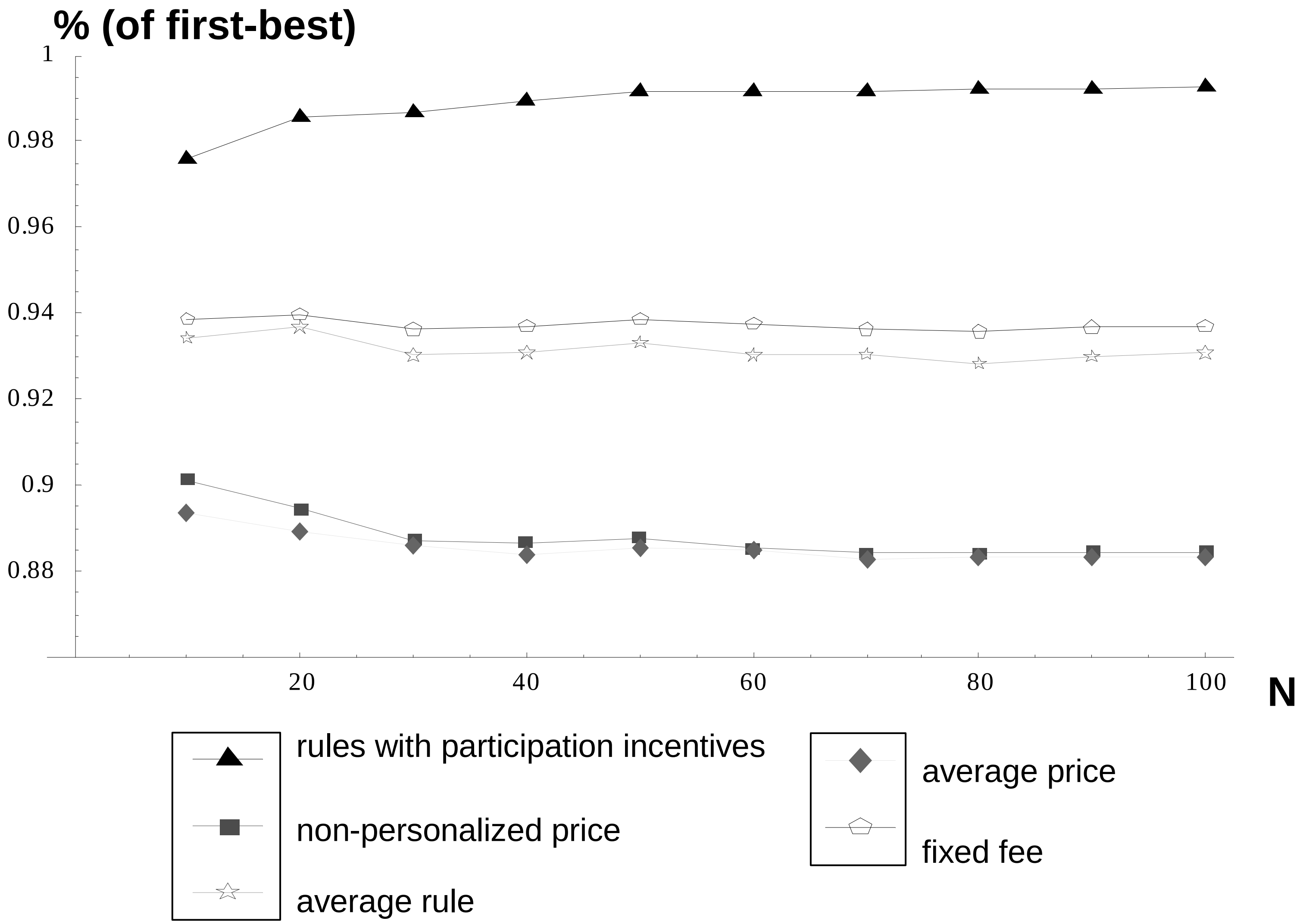
Schemes

1. First-best rule
2. First-best prices
3. Rules with participation incentives
4. Non-personalized price
5. Average rule
6. Average price
7. Fixed fee
8. Nash equilibrium

Procedure

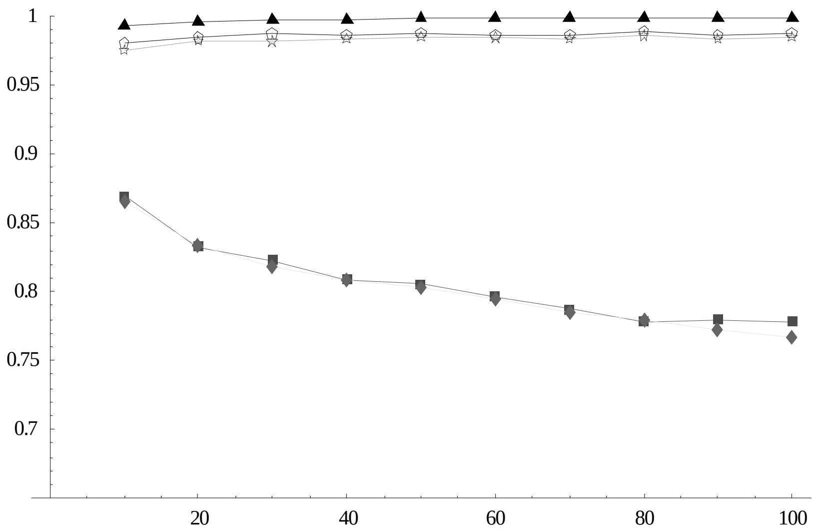
1. Fix N
2. Calculate 5 and 6
3. Draw N values $(\theta_1,\ldots,\theta_N)$
4. Calculate rules/prices in schemes 1-4
5. For this realization calculate total payoffs for all schemes
6. Repeat to step 3 and repeat 100 times
7. Average the total payoffs
8. Increase N and return to step 2

Results

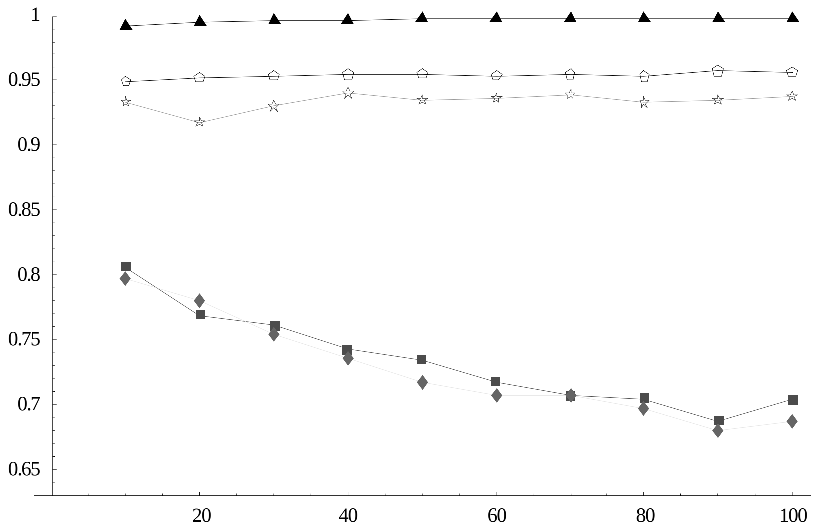


% (of first-best)
1
0.98
0.96
0.94
0.92
0.9
0.88
20
40
60
80
100
N
rules with participation incentives
non-personalized price
average rule
average price
fixed fee

The effect of heterogeneity



(μ,σ) = (0.5,0.6)



(μ,σ) = (0.5,0.8)

- The effect of variance suggests that if peers can be better classified in groups (by using some objective characteristic –like the access speed of their modem) then simple fixed fee schemes will perform better

Exercise 1

Prove that the problem

$$\max_{Q(.),\ p_1(.),\dots,p_n(.)} E[\sum \theta_i u(Q(\theta)) - c(Q(\theta))]$$

subject to feasibility, IC and IR, is equivalent to

$$\max_{Q(.)} E_\theta[\sum_1^n \theta_i u(Q(\theta)) - c(Q(\theta))]$$

such that

$$E_\theta[\sum_1^n (\theta_i - \frac{1-F(\theta_i)}{f(\theta_i)})u(Q(\theta)) - c(Q(\theta))] \geq 0$$

with the expected payments given by

$$P_i(\theta_i) = P_i(0) + \theta_i V_i(\theta_i) + \int_o^{\theta_i} V_i(y)dy$$

$$V_i(\theta_i) = E_{\theta_{-i}}[u(Q(\theta_i,\theta_{-i}))], \quad P_i(\theta_i) = E_{\theta_{-i}}[p_i(\theta_i,\theta_{-i})]$$

Exercise 2

- Prove that in the case of the bridge construction with incomplete information, the condition for the construction is

$$Q(\theta) = 1 \ \text{iff} \ \sum_1^n \theta_i - \frac{\lambda}{1+\lambda} \sum_1^n \frac{1 - F(\theta_i)}{f(\theta_i)} \geq c$$

- Suggest a way to compute the Lagrange multiplier

Exercise 3

- Extend the public good model of file sharing when files are of two types of different popularity
- How can we incorporate congestion?
- Different peer types w.r.t. cost ?

Back up slides

Nash Equilibrium

All peers maximize their payoff: $\max_{f_i} u_i(\mathrm{f})$, taken as given the files shared by all other peers

$$\frac{\partial u_i(\mathrm{f})}{\partial f_i} = \frac{\theta_i}{2\sqrt{f_i}} \upsilon'(\sum_{j=1}^{N} \sqrt{f_j}) - 1 \le 0 \qquad f_i \ge 0$$

$$\Rightarrow \quad \hat{f} = \left(\frac{\theta_i}{2} \upsilon'(\sum_{j=1}^{N} \sqrt{f_j}) \right)^2$$

Peers have the incentive to free-ride on efforts of others

Optimizing social welfare

$$SW = \sum_{i=1}^{N} (\theta_i \upsilon(\sum_{j=1}^{N} \sqrt{f_j}) - f_i)$$

$$\frac{\partial SW}{\partial f_i} = \frac{\theta_i}{2\sqrt{f_i}} \upsilon'(\sum_{j=1}^{N} \sqrt{f_j}) - 1 + \frac{\sum_{j \neq i} \theta_j}{2\sqrt{f_i}} \upsilon'(\sum_{j=1}^{N} \sqrt{f_j}) \leq 0$$

$$f^* = \left(\frac{\Theta}{2} \upsilon'(F^*) \right)^2 \qquad \Theta \equiv \sum_{j=1}^{N} \theta_j \quad \text{and} \quad F^* = N\sqrt{f^*}$$

First-best rule: Enforce all peers to contribute f^* *files*

$$p_i = \frac{\sum_{j \neq i} \theta_j}{2\sqrt{f_i}} \upsilon'(\sum_{j=1}^{N} \sqrt{f_j}) = \frac{\Theta_{-i}}{\Theta}$$

First-best price: Peer i *is paid with* p_i *per file shared*

Complete information rules with participation incentives

$$\max_{\{f_1,\ldots,f_N\}} \sum_{i=1}^{N} (\theta_i \upsilon(\sum_{j=1}^{N} \sqrt{f_j}) - f_i) \qquad \text{s.t.} \quad \boxed{\theta_i \upsilon(\sum_{j=1}^{N} \sqrt{f_j}) \geq f_i, \ \forall i}$$

$$\tilde{f}_i = \begin{cases} \theta_i \upsilon(\tilde{F}), & i < k \\ \theta_k \upsilon(\tilde{F}), & i \geq k \end{cases}$$

Choose the threshold *k* that maximizes total payoffs

Uniform complete information prices

It can be shown that for convex cost functions there is a uniform price that converges to the first best one as *N* gets large

$$p_i = \frac{\sum_{j \neq i} \theta_j}{2\sqrt{f_i}} \upsilon'\left(\sum_{j=1}^{N} \sqrt{f_j}\right) = \frac{\Theta_{-i}}{\Theta} \qquad p^* = \frac{\sum_j \theta_j}{2\sqrt{f_i}} \upsilon'\left(\sum_{j=1}^{N} \sqrt{f_j}\right) = 1!$$

In our model (due to the linear cost model) this price equals to 1. We will use alternatively ..

$$p = 1 - \frac{\max\{\theta_1, \ldots, \theta_N\}}{\Theta}$$